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A NOTE ON DIVIDEND IRRELEVANCE AND THE GORDON VALUATION MODEL*

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THE CONTRIBUTIONS of Modigliani and Miller to the theory of corporate finance are justly celebrated:[1] indeed many authorities would date the development of modern analytical financial theory to their path-breaking 1958 article. Yet, while the points of disagreement between the theory of capital structure expressed in their earlier articles and the traditional theory have been narrowed down to differing empirical assumptions, the same cannot be said of their later article on dividend policy: "Dividend Policy, Growth, and the Valuation of Shares."[2]

Nine years after the publication of this latter article there continue to co-exist among financial theorists two opposing views on the importance of dividend policy in perfect markets. The first and older view, originally articulated by Myron Gordon,[3] and still commanding widespread assent, can be paraphrased by the statement that even in perfect capital markets,[4] the existence of uncertainty about the future suffices to make the price of a share dependent upon the dividend policy which is followed: and that in particular, the more generous is the dividend policy, the higher will be the price of the share. Miller and Modigliani on the other hand, have argued that once the investment policy of a firm is given, the price of its shares is invariant with respect to the size of the dividend paid.[5]

The issue between these opposing views cannot be settled by resort to experience, for the fundamental reason that the above hypotheses relate to the effects of dividend policy in perfect capital markets, whereas of course actual securities markets suffer from several imperfections, the most important of which, from the point of view of dividend policy, are the existence of transactions costs and of differential taxes on income from dividends and capital gains.[6] Despite this, there is a paucity of articles on the theoretical differences

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1. See Modigliani and Miller [8, 9, 10].

2. Modigliani and Miller [8].

3. Gordon [2, 3, 4].

4. The perfection of capital markets in this sense is usually taken to exclude such factors as taxes and transactions costs. It is further assumed that all market participants have access to the same information, though of course the existence of uncertainty precludes the possession of perfect information about the future.

5. Except insofar as current dividends may carry information about the future prospects of the firm.

6. Both of these factors may cause investors to have specific preferences between the two income forms. For a summary of other difficulties in empirical tests of investor attitudes towards dividends see Friend and Puckett [1].

between Gordon and M-M,[7] and the textbook treatment of the two conflicting theories is in most cases highly unsatisfactory.[8] The student of finance is thus left in a quandary: both authors develop plausible theories from reasonable assumptions, but mystifyingly reach opposing conclusions.

This paper does not aim to break new ground but rather to clarify the main points at issue between the two theories; first, by showing that the Gordon argument does in fact rest upon a confounding of the effects of dividend policy and investment policy; and secondly, by showing that the M-M dividend irrelevance theorem can be derived from a somewhat weaker assumption than that of symmetric market rationality.

Gordon's discussion of dividend policy develops directly from his stock price valuation model, which asserts that the price of a share is equal to the discounted value of expected future dividends.[9] If dividends are expected to grow at the constant rate, g, in perpetuity, and the discount rate is k, this principle leads to the familiar valuation formula:

$$P = \frac{D}{k - g} \tag{1}$$

where P is the current stock price, and D is the current amount of dividends per share.

If the firm retains a constant fraction, b, of its earnings per share, and earns a constant average rate of return, r, on its investment, employing no outside financing, then

$$g = br$$

and

$$D = (1 - b)Y$$

where Y is the current amount of earnings per share. Substitution for g and D in (1) yields

$$P = \frac{(1 - b)Y}{k - br}. \tag{2}$$

Equation (2) may then be used to evaluate the effect of alternative retention ratios (and therefore payout ratios) on the value of a share. In particular, if the discount rate, k, and the average rate of return on investment, r, are independent of the retention ratio, b, then the effect of alternative retention ratios may be examined by differentiating (2) partially with respect to b:

7. However, see Walter [14].

8. Weston and Brigham [15, P482] for example reach the rather surprising conclusion "that all the approaches to dividends (including those of Gordon and M-M) result in the same policy implications as the Walter formulation." Van Horne [13, P185] summarizes the arguments of Gordon and M-M, before passing on to questions of imperfections in capital markets, but offers the reader no guide as to how to choose between them. Mao [6, P484] leaves the question open, stating that "The cause of this conflict between the dividend (Gordon) and the earnings theorists (M-M) can be traced to their differing assumptions concerning the effect of dividend policy on the rate of return required by investors."

9. This presentation of Gordon's model relies mainly on Gordon [3].

$$\frac{\partial P}{\partial b} = \frac{(r - k)Y}{(k - br)^2}. \tag{3}$$

It follows that the condition for the share price to be invariant with respect to the fraction of earnings retained and re-invested is just that the average rate of return, r, which is also the rate of return on the marginal investment of retained earnings, be equal to the discount rate, k. It should be observed that in this case the marginal investment has a zero net present value, so that Gordon has shown that if the effects of investment policy are neutralized by setting the net present value of marginal investment equal to zero, then dividend policy is irrelevant. The similarity of this result to that obtained by M-M may be noted:[10] M-M neutralized the effects of investment policy by holding the amount of investment constant under alternative dividend policies, whereas Gordon has achieved the same result by holding the net present value of investment constant.

However, Gordon goes on to argue that, contrary to what has been assumed so far, it is not in general the case that the discount rate, k, is independent of the retention ratio, b. The rationale for this is that dividends expected at different dates in the future will be subject to different risks, and that therefore each expected future dividend should be discounted at a different rate to reflect this differential risk.[11] Thus the valuation formula for a share should be written explicitly as:

$$P = \frac{D(1)}{(1 + k_1)} + \frac{D(2)}{(1 + k_2)^2} + .. \frac{D(t)}{(1 + k_t)^t} + \ldots \tag{4}$$

where:

$D(t)$ = the expected dividend per share t periods in the future,
k_t = the discount rate appropriate to the risk of a dividend expected t periods ahead.

As M-M point out,[12] since (4) gives the market value of a share, which is determined by the interactions of the supply and demand functions of all market participants, the k_t's should strictly be interpreted as market determined discount rates, rather than the subjective discount rates of any individual investor, although these will be equal in equilibrium.

Gordon admits that it is not possible a priori to determine whether k_t is an increasing or decreasing function of t. But, he observes, "The important point to note, however, is that there is nothing to guarantee that k_t is a constant for all values of t."[13]

10. However, it should be observed that M-M's proof of dividend irrelevance under uncertainty proceeds in a much more general framework, which requires no assumptions about the way in which investors evaluate future dividends, whereas Gordon explicitly assumes that they are discounted.

11. Robichek and Myers [12] have argued for the use of certainty equivalents rather than adjustments to the discount rate to account for risk; however, when a different discount rate is assigned to each dividend payment, the two approaches are equivalent.

12. M-M [8, P424] footnote 19.

13. Gordon [3, P43].

Once it is admitted that k_t is a function of t, then it follows that k in the valuation formula (1) is a generalized average of the individual k_t, whose weights are not independent of the time path of expected dividends.[14] In the particular case of exponential dividend growth considered here, k is a function of the growth rate g. i.e. $\frac{\partial k}{\partial g} \neq 0$.

Since the growth rate of dividends is given by $g = br$, it follows that if r is independent of b,

$$\frac{\partial k}{\partial b} = \frac{\partial k}{\partial g} \cdot \frac{\partial g}{\partial b}$$

$$= \frac{\partial k}{\partial g} \cdot r \neq 0. \tag{5}$$

Thus, allowing for this dependence of k on b, the effect of a change in the retention ratio on share price is given by:

$$\frac{dP}{db} = \frac{Y\left[r - k - (1-b)\frac{\partial k}{\partial b}\right]}{(k-br)^2}. \tag{6}$$

Then, if $r = k$

$$\frac{dP}{db} = \frac{-Y(1-b)\frac{\partial k}{\partial b}}{(k-br)^2} \neq 0.$$

So, even if the rate of return on the marginal investment, r, is set equal to the average discount rate, k, the price of a share is not independent of the retention ratio, unless $\frac{\partial k}{\partial b} = 0$. Gordon concludes from this that, in general, stock price and the cost of capital depend on dividend policy.[15]

M-M have asserted that Gordon's argument rests on a confounding of the effects of dividend and investment policy;[16] Gordon, however, has rejected this argument.[17] What does this confounding of the two effects mean? It means presumably that the change in the amount of investment which accompanies the change in dividend policy in Gordon's model would of itself have effected a change in share price, regardless of how it was financed, and that

14. For a mathematical description of this dependence see the Appendix to Gordon [3] written by Gangolli.

15. "Therefore the statement that a corporation's cost of capital is independent of its dividend rate, or that dividend policy has no influence on share price, implies that k is independent of br." [6, P87]

16. "For all its ingenuity, however, and its seeming foundation in uncertainty, the argument clearly suffers fundamentally from the typical confounding of dividend policy with investment policy that so frequently accompanies use of the internal financing model." [11, P425]

17. "It is well-known that when the rate of return on investment is set equal to the discount rate, changing the level of investment has no effect on share price. By this means I neutralized the profitability of investment. It seems to me perfectly clear that I did not confound investment and dividend policy: I changed the discount rate." [5, P265]

Gordon is mistakenly attributing to dividend policy the effect of the change in investment policy: in other words, M-M are disputing that the net present value of the marginal investment is zero, even when r is set equal to k. The validity of the M-M argument is readily examined by considering the net present value of the marginal investment implied by a change in the retention ratio in Gordon's model.

Observe first that a change in the retention ratio from b_1 to b_2 involves not just a single change in the amount invested and earned, but a change in earnings and investment in all subsequent periods, or a change in investment policy. It is the net present value of this change in investment policy which must be evaluated.

Denote by ΔI_t the change in period t investment implied by a change in the retention ratio from b_1 to b_2.[18] Then, given that the marginal investment earns a perpetual rate of return, r, and that cash flows expected t periods ahead are discounted at the rate k_t,[19] the net present value of the change in investment policy is given by:

$$\text{NPV} = \sum_{t=1}^{\infty} \frac{\Delta I_t}{(1+k_t)^t} \left(-1 + r \sum_{\tau=1}^{\infty} \frac{1}{(1+k_{t+\tau})^\tau} \right). \tag{7}$$

Now, in general, setting $r = k$ will not equate the expression in (7) to zero, unless all the k_t are equal, and equal to k—the only circumstance in which Gordon finds dividend policy to be irrelevant!

Thus it appears that Gordon has been misled by the fact that in the special case when all the k_t are equal, setting $r = k$ does neutralize the effect of the profitability of investment, into believing that this procedure neutralized the profitability of investment when the k_t are unequal.[20]

Therefore M-M's argument is upheld: Gordon's proof of the relevance of dividend policy does rest on a confounding of investment and dividend policy effects. It is true of course that changing the firm's dividend policy will change the average discount rate in the valuation formula (1). This is not to say, however, that it changes the corporation's cost of capital, for as should be clear by now, if the k_t are unequal, the corporation has no unique cost of capital: the average discount rate applicable to an investment project will depend upon the exact time path of the project's returns.[21] Thus k in the

18. ΔI_t is given by:

$$\Delta I_t = Y[b_2(1+b_2r)^t - b_1(1+b_1r)^t].$$

19. There is some awkwardness in assuming that cash outflows for investment, which are discretionary, should be discounted at the same rate as the resulting cash inflows, which are not. However, this assumption appears to be compatible with Gordon's practice, for he assumes that the discount rate applicable to a dividend depends only on its futurity and not on the magnitude of earnings and retentions in that period. This is one of the difficulties inherent in a discounted cash flow approach to valuation under uncertainty.

20. It is worth noting that the k_t will be unequal even under conditions of certainty, if the term structure of interest rates is not flat. Thus Gordon's proof of dividend relevance, if it were valid, would apply even under conditions of certainty!

21. For this reason it would appear preferable to employ the net present value approach to investment decisions, for this does not require specification of a single discount rate.

valuation formula (1), far from being the corporation's cost of capital, is but an algebraic artefact, and as such should be irrelevant for decision-making purposes.

M-M's proof of dividend irrelevance under uncertainty proceeds by way of the familiar arbitrage argument. Assuming that investment policy is held constant, M-M show that it follows from the firm's budget constraint that the total return to shareholders in firm i, $R_i(0)$,[22] from holding the shares of the firm for one period is:

$$\tilde{R}_i(0) = \tilde{X}_i(0) - \tilde{I}_i(0) + \tilde{V}_i(1) \tag{8}$$

where $X_i(0)$ and $I_i(0)$ are the firm's operating income stream and investment budget for the period, and $V_i(1)$ is its value at the end of the period. M-M posit two firms ($i = 1, 2$) identical in all respects except their first period dividend, and argue that:

(1) $\tilde{R}_1(0) = \tilde{R}_2(0)$, and therefore,
(2) the initial values of the firms are identical.

(1) Since the two firms are assumed identical except for their first period dividend policies, it follows by assumption that $X_1(0) = X_2(0)$ and $I_1(0) = I_2(0)$. They argue moreover that all investors will expect that $V_1(1) = V_2(1)$.

This argument derives from their assumption of symmetric market rationality, which requires first that every market participant behaves rationally in the sense of preferring more wealth to less and being indifferent to the form in which his wealth increment is received; and secondly that, in forming his expectations, he believes that every other market participant both behaves rationally and believes that all others also behave rationally. Thus this assumption implies that all participants will believe that the two firms will be valued rationally at the end of the period so that $V_1(1)$ and $V_2(1)$ will depend only on the prospective future earnings, dividends and investment of the two firms from period 1. Since these are identical by assumption, it follows that all investors will expect $V_1(1) \equiv V_2(1)$, so that $R_1(0) \equiv R_2(0)$.

A more direct set of assumptions leading to the same conclusion might be called the "independence of irrelevant information" which requires that:

(a) investors are rational in the above sense, and
(b) shares are valued only on the basis of their future prospects, and
(c) at least some investors know from experience that this is so.

It follows directly from this set of assumptions that since the prospects of the two firms are known to be indentical as of the end of the period, at least some investors will expect $\tilde{V}_1(1) \equiv \tilde{V}_2(1)$. Assumption (a) is the standard assumption of rational behavior; assumption (b), in addition to being plausible, has the advantage of familiarity, for it is an implicit assumption of all stock valuation models (including Gordon's), while assumption (c) would appear to be empirically valid.

(2) On the assumption of symmetric market rationality, all investors will

22. Note that "return" refers here, not to the rate of return, but to the total cash receipts of investors including their initial investment.

perceive that $\tilde{R}_1(0) \equiv \tilde{R}_2(0)$, and therefore by the assumption of individual rationality will value the two firms equally, so that the value of the firm is independent of its first period dividend policy.

On the weaker assumption of the independence of irrelevant information, at least some traders who realize that $\tilde{R}_1(0) \equiv \tilde{R}_2(0)$ will arbitrage away any difference in the initial valuation of the two firms, leading to the same result.

Having shown that first period dividend policy is irrelevant, M-M proceed to show that $\tilde{V}_i(2)$ and hence $\tilde{V}_i(1)$ and $\tilde{V}_i(0)$ are independent of second period dividend policy, and thence by induction that the value of the firm is independent of its dividend policy in all subsequent periods, once investment policy is given.

Thus any denial of the irrelevance of dividend policy must rely upon a rejection of the principle of symmetric market rationality, and the assumption of the independence of irrelevant information. To reject the latter assumption requires one of the following three assertions: either that:

(a) investors are not rational, or
(b) stock prices depend on past events as well as on their expected future prospects, or
(c) there exist no investors who understand the security valuation process.

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