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DIVIDEND POLICY, GROWTH, AND THE VALUATION OF SHARES*

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The effect of a firm's dividend policy on the current price of its shares is a matter of considerable importance, not only to the corporate officials who must set the policy, but to investors planning portfolios and to economists seeking to understand and appraise the functioning of the capital markets. Do companies with generous distribution policies consistently sell at a premium over those with niggardly payouts? Is the reverse ever true? If so, under what conditions? Is there an optimum payout ratio or range of ratios that maximizes the current worth of the shares?

Although these questions of fact have been the subject of many empirical studies in recent years no consensus has yet been achieved. One reason appears to be the absence in the literature of a complete and reasonably rigorous statement of those parts of the economic theory of valuation bearing directly on the matter of dividend policy. Lacking such a statement, investigators have not yet been able to frame their tests with sufficient precision to distinguish adequately between the various contending hypotheses. Nor have they been able to give a convincing explanation of what their test results do imply about the underlying process of valuation.

In the hope that it may help to overcome these obstacles to effective empirical testing, this paper will attempt to fill the existing gap in the theoretical literature on valuation. We shall begin, in Section I, by examining the effects of differences in dividend policy on the current price of shares in an ideal economy characterized by perfect capital markets, rational behavior, and perfect certainty. Still within this convenient analytical framework we shall go on in Sections II and III to consider certain closely related issues that appear to have been responsible for considerable misunderstanding of the role of dividend policy. In particular, Section II will focus on the long-standing debate about what investors "really" capitalize when they buy shares; and Section III on the much mooted relations between price, the rate of growth of

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profits, and the rate of growth of dividends per share. Once these fundamentals have been established, we shall proceed in Section IV to drop the assumption of certainty and to see the extent to which the earlier conclusions about dividend policy must be modified. Finally, in Section V, we shall briefly examine the implications for the dividend policy problem of certain kinds of market imperfections.

I. EFFECT OF DIVIDEND POLICY WITH PERFECT MARKETS, RATIONAL BEHAVIOR, AND PERFECT CERTAINTY

The meaning of the basic assumptions.—Although the terms "perfect markets," "rational behavior," and "perfect certainty" are widely used throughout economic theory, it may be helpful to start by spelling out the precise meaning of these assumptions in the present context.

1. In "perfect capital markets," no buyer or seller (or issuer) of securities is large enough for his transactions to have an appreciable impact on the then ruling price. All traders have equal and costless access to information about the ruling price and about all other relevant characteristics of shares (to be detailed specifically later). No brokerage fees, transfer taxes, or other transaction costs are incurred when securities are bought, sold, or issued, and there are no tax differentials either between distributed and undistributed profits or between dividends and capital gains.

2. "Rational behavior" means that investors always prefer more wealth to less and are indifferent as to whether a given increment to their wealth takes the form of cash payments or an increase in the market value of their holdings of shares.

3. "Perfect certainty" implies complete assurance on the part of every investor as to the future investment program and the future profits of every corporation. Because of this assurance, there is, among other things, no need to distinguish between stocks and bonds as sources of funds at this stage of the analysis. We can, therefore, proceed as if there were only a single type of financial instrument which, for convenience, we shall refer to as shares of stock.

The fundamental principle of valuation.—Under these assumptions the valuation of all shares would be governed by the following fundamental principle: the price of each share must be such that the rate of return (dividends plus capital gains per dollar invested) on every share will be the same throughout the market over any given interval of time. That is, if we let

$d_j(t)$ = dividends per share paid by firm j during period t

$p_j(t)$ = the price (ex any dividend in $t-1$) of a share in firm j at the start of period t,

we must have

$$\frac{d_j(t)+p_j(t+1)-p_j(t)}{p_j(t)} = \rho(t) \text{ independent of } j\,; \tag{1}$$

or, equivalently,

$$p_j(t)=\frac{1}{1+\rho(t)}[d_j(t)+p_j(t+1)] \tag{2}$$

for each j and for all t. Otherwise, holders of low-return (high-priced) shares could increase their terminal wealth by selling these shares and investing the proceeds in shares offering a higher rate of return. This process would tend to drive down the prices of the low-return shares and drive up the prices of high-return shares until the differential in rates of return had been eliminated.

The effect of dividend policy.—The implications of this principle for our problem of dividend policy can be seen somewhat more easily if equation (2) is restated in terms of the value of the enterprise as a whole rather than in terms of the value of an individual share. Dropping the firm subscript j since this will lead to no ambiguity in the present context and letting

$n(t)$ = the number of shares of record at the start of t

$m(t+1)$ = the number of new shares (if any) sold during t at the ex dividend closing price $p(t+1)$, so that

$n(t+1) = n(t) + m(t+1)$

$V(t) = n(t)\,p(t)$ = the total value of the enterprise and

$D(t) = n(t)\,d(t)$ = the total dividends paid during t to holders of record at the start of t,

we can rewrite (2)

$$V(t) = \frac{1}{1+\rho(t)}\left[D(t) + n(t)\,p(t+1)\right]$$

$$= \frac{1}{1+\rho(t)}\left[D(t) + V(t+1) - m(t+1)\,p(t+1)\right]. \quad (3)$$

The advantage of restating the fundamental rule in this form is that it brings into sharper focus the three possible routes by which current dividends might affect the current market value of the firm $V(t)$, or equivalently the price of its individual shares, $p(t)$. Current dividends will clearly affect $V(t)$ via the first term in the bracket, $D(t)$. In principle, current dividends might also affect $V(t)$ indirectly via the second term, $V(t+1)$, the new ex dividend market value. Since $V(t+1)$ must depend only on future and not on past events, such could be the case, however, only if both (*a*) $V(t+1)$ were a function of future dividend policy and (*b*) the current distribution $D(t)$ served to convey some otherwise unavailable information as to what that future dividend policy would be. The first possibility being the relevant one from the standpoint of assessing the effects of dividend policy, it will clarify matters to assume, provisionally, that the future dividend policy of the firm is known and given for $t+1$ and all subsequent periods and is independent of the actual dividend decision in t. Then $V(t+1)$ will also be independent of the current dividend decision, though it may very well be affected by $D(t+1)$ and all subsequent distributions. Finally, current dividends can influence $V(t)$ through the third term, $-m(t+1)\,p(t+1)$, the value of new shares sold to outsiders during the period. For the higher the dividend payout in any period the more the new capital that must be raised from external sources to maintain any desired level of investment.

The fact that the dividend decision effects price not in one but in these two conflicting ways—directly via $D(t)$ and inversely via $-m(t)\,p(t+1)$—is, of course, precisely why one speaks of there being a dividend policy *problem*. If the firm raises its dividend in t, given its investment decision, will the increase in the cash payments to the current holders be more or less than enough to offset their lower share of the terminal value? Which is the better strategy for the firm in financing the investment: to reduce dividends and rely on retained earnings or to raise dividends but float more new shares?

In our ideal world at least these and related questions can be simply and immediately answered: the two dividend effects must always exactly cancel out so that the payout policy to be followed in t will have *no* effect on the price at t.

We need only express $m(t+1) \cdot p(t+1)$ in terms of $D(t)$ to show that such must indeed be the case. Specifically, if $I(t)$ is the given level of the firm's investment or increase in its holding of physical assets in t and if $X(t)$ is the firm's total net profit for the period, we know that the amount of outside capital required will be

$$m(t+1)p(t+1) = I(t) - [X(t) - D(t)]. \quad (4)$$

Substituting expression (4) into (3), the $D(t)$ cancel and we obtain for the value of the firm as of the start of t

$$V(t) \equiv n(t)p(t) = \frac{1}{1+\rho(t)}[X(t) - I(t) + V(t+1)]. \quad (5)$$

Since $D(t)$ does not appear directly among the arguments and since $X(t)$, $I(t)$, $V(t+1)$ and $\rho(t)$ are all independent of $D(t)$ (either by their nature or by assumption) it follows that the current value of the firm must be independent of the current dividend decision.

Having established that $V(t)$ is unaffected by the current dividend decision it is easy to go on to show that $V(t)$ must also be unaffected by any future dividend decisions as well. Such future decisions can influence $V(t)$ only via their effect on $V(t+1)$. But we can repeat the reasoning above and show that $V(t+1)$—and hence $V(t)$—is unaffected by dividend policy in $t+1$; that $V(t+2)$—and hence $V(t+1)$ and $V(t)$—is unaffected by dividend policy in $t+2$; and so on for as far into the future as we care to look. Thus, we may conclude that given a firm's investment policy, the dividend payout policy it chooses to follow will affect neither the current price of its shares nor the total return to its shareholders.

Like many other propositions in economics, the irrelevance of dividend policy, given investment policy, is "obvious, once you think of it." It is, after all, merely one more instance of the general principle that there are no "financial illusions" in a rational and perfect economic environment. Values there are determined solely by "real" considerations—in this case the earning power of the firm's assets and its investment policy—and not by how the fruits of the earning power are "packaged" for distribution.

Obvious as the proposition may be, however, one finds few references to it in the extensive literature on the problem.[1] It is true that the literature abounds with statements that in some "theoretical" sense, dividend policy ought not to count; but either that sense is not clearly specified or, more frequently and especially among economists, it is (wrongly) identified with a situation in which the firm's internal rate of return is the same as the external or market rate of return.[2]

A major source of these and related misunderstandings of the role of the dividend policy has been the fruitless concern and controversy over what investors "really" capitalize when they buy shares. We say fruitless because as we shall now proceed to show, it is actually possible to derive from the basic principle of valuation (1) not merely one, but several valuation formulas each starting from one of the "classical" views of what is being capitalized by investors. Though differing somewhat in outward appearance, the various formulas can be shown to be equivalent in all essential respects including, of course, their implication that dividend policy is irrelevant. While the

[1] Apart from the references to it in our earlier papers, especially [16], the closest approximation seems to be that in Bodenborn [1, p. 492], but even his treatment of the role of dividend policy is not completely explicit. (The numbers in brackets refer to references listed below, pp. 432–33).

[2] See below p. 424.

controvery itself thus turns out to be an empty one, the different expressions do have some intrinsic interest since, by highlighting different combinations of variables they provide additional insights into the process of valuation and they open alternative lines of attack on some of the problems of empirical testing.

II. WHAT DOES THE MARKET "REALLY" CAPITALIZE?

In the literature on valuation one can find at least the following four more or less distinct approaches to the valuation of shares: (1) the discounted cash flow approach; (2) the current earnings plus future investment opportunities approach; (3) the stream of dividends approach; and (4) the stream of earnings approach. To demonstrate that these approaches are, in fact, equivalent it will be helpful to begin by first going back to equation (5) and developing from it a valuation formula to serve as a point of reference and comparison. Specifically, if we assume, for simplicity, that the market rate of yield $\rho(t) = \rho$ for all t,[3] then, setting $t = 0$, we can rewrite (5) as

$$V(0) = \frac{1}{1+\rho}[X(0) - I(0)] + \frac{1}{1+\rho} V(1). \quad (6)$$

Since (5) holds for all t, setting $t = 1$ permits us to express $V(1)$ in terms of $V(2)$ which in turn can be expressed in terms of $V(3)$ and so on up to any arbitrary terminal period T. Carrying out these substitutions, we obtain

$$V(0) = \sum_{t=0}^{T-1} \frac{1}{(1+\rho)^{t+1}}[X(t) - I(t)] + \frac{1}{(1+\rho)^{T}} V(T). \quad (7)$$

In general, the remainder term $(1+\rho)^{-T} \cdot V(T)$ can be expected to approach zero as T approaches infinity[4] so that (7) can be expressed as

$$V(0) = \lim_{T\to\infty} \sum_{t=0}^{T-1} \frac{1}{(1+\rho)^{t+1}} \times [X(t) - I(t)], \quad (8)$$

which we shall further abbreviate to

$$V(0) = \sum_{t=0}^{\infty} \frac{1}{(1+\rho)^{t+1}}[X(t) - I(t)]. \quad (9)$$

The discounted cash flow approach.—Consider now the so-called discounted cash flow approach familiar in discussions of capital budgeting. There, in valuing any specific machine we discount at the market rate of interest the stream of cash receipts generated by the machine; plus any scrap or terminal value of the machine; and minus the stream of cash outlays for direct labor, materials, repairs, and capital additions. The same approach, of course, can also be applied to the firm as a whole which may be thought of in this context as simply a large, composite machine.[5] This ap-

[3] More general formulas in which $\rho(t)$ is allowed to vary with time can always be derived from those presented here merely by substituting the cumbersome product

$$\prod_{\tau=0}^{t} [1+\rho(\tau)] \quad \text{for} \quad (1+\rho)^{t+1}.$$

[4] The assumption that the remainder vanishes is introduced for the sake of simplicity of exposition only and is in no way essential to the argument. What is essential, of course, is that $V(0)$, i.e., the sum of the two terms in (7), be finite, but this can always be safely assumed in economic analysis. See below, n. 14.

[5] This is, in fact, the approach to valuation normally taken in economic theory when discussing the value of the *assets* of an enterprise, but much more rarely applied, unfortunately, to the value of the liability side. One of the few to apply the approach to the shares as well as the assets is Bodenhorn in [1], who uses it to derive a formula closely similar to (9) above.

proach amounts to defining the value of the firm as

$$V(0) = \sum_{t=0}^{T-1} \frac{1}{(1+\rho)^{t+1}} \times [\mathcal{R}(t) - \mathcal{O}(t)] + \frac{1}{(1+\rho)^T} V(T), \quad (10)$$

where $\mathcal{R}(t)$ represents the stream of cash receipts and $\mathcal{O}(t)$ of cash outlays, or, abbreviating, as above, to

$$V(0) = \sum_{t=0}^{\infty} \frac{1}{(1+\rho)^{t+1}} [\mathcal{R}(t) - \mathcal{O}(t)]. \quad (11)$$

But we also know, by definition, that $[X(t) - I(t)] = [\mathcal{R}(t) - \mathcal{O}(t)]$ since, $X(t)$ differs from $\mathcal{R}(t)$ and $I(t)$ differs from $\mathcal{O}(t)$ merely by the "cost of goods sold" (and also by the depreciation expense if we wish to interpret $X(t)$ and $I(t)$ as net rather than gross profits and investment). Hence (11) is formally equivalent to (9), and the discounted cash flow approach is thus seen to be an implication of the valuation principle for perfect markets given by equation (1).

The investment opportunities approach.—Consider next the approach to valuation which would seem most natural from the standpoint of an investor proposing to buy out and operate some already-going concern. In estimating how much it would be worthwhile to pay for the privilege of operating the firm, the amount of dividends to be paid is clearly not relevant, since the new owner can, within wide limits, make the future dividend stream whatever he pleases. For him the worth of the enterprise, as such, will depend only on: (*a*) the "normal" rate of return he can earn by investing his capital in securities (i.e., the market rate of return); (*b*) the earning power of the physical assets currently held by the firm; and (*c*) the opportunities, if any, that the firm offers for making additional investments in real assets that will yield more than the "normal" (market) rate of return. The latter opportunities, frequently termed the "good will" of the business, may arise, in practice, from any of a number of circumstances (ranging all the way from special locational advantages to patents or other monopolistic advantages).

To see how these opportunities affect the value of the business assume that in some future period t the firm invests $I(t)$ dollars. Suppose, further, for simplicity, that starting in the period immediately following the investment of the funds, the projects produce net profits at a constant rate of $\rho^*(t)$ per cent of $I(t)$ in each period thereafter.[6] Then the present worth as of t of the (perpetual) stream of profits generated will be $I(t)\,\rho^*(t)/\rho$, and the "good will" of the projects (i.e., the difference between worth and cost) will be

$$I(t)\frac{\rho^*(t)}{\rho} - I(t) = I(t)\left[\frac{\rho^*(t) - \rho}{\rho}\right].$$

The present worth as of now of this future "good will" is

$$I(t)\left[\frac{\rho^*(t) - \rho}{\rho}\right](1+\rho)^{-(t+1)},$$

and the present value of all such future opportunities is simply the sum

$$\sum_{t=0}^{\infty} I(t)\frac{\rho^*(t) - \rho}{\rho}(1+\rho)^{-(t+1)}.$$

Adding in the present value of the (uniform perpetual) earnings, $X(0)$, on the as-

[6] The assumption that $I(t)$ yields a uniform perpetuity is not restrictive in the present certainty context since it is always possible by means of simple, present-value calculations to find an equivalent uniform perpetuity for any project, whatever the time shape of its actual returns. Note also that $\rho^*(t)$ is the *average* rate of return. If the managers of the firm are behaving rationally, they will, of course, use ρ as their cut-off criterion (cf. below p. 418). In this event we would have $\rho^*(t) \geq \rho$. The formulas remain valid, however, even where $\rho^*(t) < \rho$.

sets currently held, we get as an expression for the value of the firm

$$V(0) = \frac{X(0)}{\rho} + \sum_{t=0}^{\infty} I(t) \times \frac{\rho^*(t) - \rho}{\rho}(1+\rho)^{-(t+1)}. \quad (12)$$

To show that the same formula can be derived from (9) note first that our definition of $\rho^*(t)$ implies the following relation between the $X(t)$:

$$X(1) = X(0) + \rho^*(0)I(0),$$

$$\cdots\cdots\cdots\cdots\cdots\cdots$$

$$X(t) = X(t-1) + \rho^*(t-1)I(t-1)$$

and by successive substitution

$$X(t) = X(0) + \sum_{\tau=0}^{t-1} \rho^*(\tau)I(\tau), \quad t = 1, 2 \ldots \infty .$$

Substituting the last expression for $X(t)$ in (9) yields

$$V(0) = [X(0) - I(0)](1+\rho)^{-1} + \sum_{t=1}^{\infty}\left[X(0) + \sum_{\tau=0}^{t-1}\rho^*(\tau)I(\tau) - I(t)\right](1+\rho)^{-(t+1)}$$

$$= X(0)\sum_{t=1}^{\infty}(1+\rho)^{-t} - I(0)(1+\rho)^{-1} + \sum_{t=1}^{\infty}\left[\sum_{\tau=0}^{t-1}\rho^*(\tau)I(\tau) - I(t)\right] \times (1+\rho)^{-(t+1)}$$

$$= X(0)\sum_{t=1}^{\infty}(1+\rho)^{-t} + \sum_{t=1}^{\infty}\left[\sum_{\tau=0}^{t-1}\rho^*(\tau)I(\tau) - I(t-1) \times (1+\rho)\right](1+\rho)^{-(t+1)}.$$

The first expression is, of course, simply a geometric progression summing to $X(0)/\rho$, which is the first term of (12). To simplify the second expression note that it can be rewritten as

$$\sum_{t=0}^{\infty} I(t)\left[\rho^*(t)\sum_{\tau=t+2}^{\infty}(1+\rho)^{-\tau} - (1+\rho)^{-(t+1)}\right].$$

Evaluating the summation within the brackets gives

$$\sum_{t=0}^{\infty} I(t)\left[\rho^*(t)\frac{(1+\rho)^{-(t+1)}}{\rho} - (1+\rho)^{-(t+1)}\right]$$

$$= \sum_{t=0}^{\infty} I(t)\left[\frac{\rho^*(t)-\rho}{\rho}\right](1+\rho)^{-(t+1)},$$

which is precisely the second term of (12).

Formula (12) has a number of revealing features and deserves to be more widely used in discussions of valuation.[7] For one thing, it throws considerable light on the meaning of those much abused terms "growth" and "growth stocks." As can readily be seen from (12), a corporation does not become a "growth stock" with a high price-earnings ratio merely because its assets and earnings are growing over time. To enter the glamor category, it is also necessary that $\rho^*(t) > \rho$. For if $\rho^*(t) = \rho$, then however large the growth in assets may be, the second term in (12) will be zero and the firm's price-earnings ratio would not rise above a humdrum $1/\rho$. The essence of "growth," in short, is not expansion, but the existence of opportunities to invest significant quantities of funds at higher than "normal" rates of return.

[7] A valuation formula analogous to (12) though derived and interpreted in a slightly different way is found in Bodenhorn [1]. Variants of (12) for certain special cases are discussed in Walter [20].

Notice also that if $\rho^*(t) < \rho$, investment in real assets by the firm will actually reduce the current price of the shares. This should help to make clear among other things, why the "cost of capital" to the firm is the same regardless of how the investments are financed or how fast the firm is growing. The function of the cost of capital in capital budgeting is to provide the "cut-off rate" in the sense of the minimum yield that investment projects must promise to be worth undertaking from the point of view of the current owners. Clearly, no proposed project would be in the interest of the current owners if its yield were expected to be less than ρ since investing in such projects would reduce the value of their shares. In the other direction, every project yielding more than ρ is just as clearly worth undertaking since it will necessarily enhance the value of the enterprise. Hence, the cost of capital or cut-off criterion for investment decisions is simply ρ.[8]

Finally, formula (12) serves to emphasize an important deficiency in many recent statistical studies of the effects of dividend policy (such as Walter [19] or Durand [4, 5]). These studies typically involve fitting regression equations in which price is expressed as some function of current earnings and dividends. A finding that the dividend coefficient is significant—as is usually the case—is then interpreted as a rejection of the hypothesis that dividend policy does not affect valuation.

Even without raising questions of bias in the coefficients,[9] it should be apparent that such a conclusion is unwarranted since formula (12) and the analysis underlying it imply only that dividends will not count given current earnings *and growth potential*. No general prediction is made (or can be made) by the theory about what will happen to the dividend coefficient if the crucial growth term is omitted.[10]

The stream of dividends approach.—From the earnings and earnings opportunities approach we turn next to the dividend approach, which has, for some reason, been by far the most popular one in the literature of valuation. This approach too, properly formulated, is an entirely valid one though, of course, not the only valid approach as its more enthusiastic proponents frequently suggest.[11] It does, however, have the disadvantage in contrast with previous approaches of obscuring the role of dividend policy. In particular, uncritical use of the

[8] The same conclusion could also have been reached, of course, by "costing" each particular source of capital funds. That is, since ρ is the going market rate of return on equity any new shares floated to finance investment must be priced to yield ρ; and withholding funds from the stockholders to finance investment would deprive the holders of the chance to earn ρ on these funds by investing their dividends in other shares. The advantage of thinking in terms of the cost of capital as the cut-off criterion is that it minimizes the danger of confusing "costs" with mere "outlays."

[9] The serious bias problem in tests using current reported earnings as a measure of $X(0)$ was discussed briefly by us in [16].

[10] In suggesting that recent statistical studies have not controlled adequately for growth we do not mean to exempt Gordon in [8] or [9]. It is true that his tests contain an explicit "growth" variable, but it is essentially nothing more than the ratio of retained earnings to book value. This ratio would not in general provide an acceptable approximation to the "growth" variable of (12) in any sample in which firms resorted to external financing. Furthermore, even if by some chance a sample was found in which all firms relied entirely on retained earnings, his tests then could not settle the question of dividend policy. For if all firms financed investment internally (or used external financing only in strict proportion to internal financing as Gordon assumes in [8]) then there would be no way to distinguish between the effects of dividend policy and investment policy (see below p. 424).

[11] See, e.g., the classic statement of the position in J. B. Williams [21]. The equivalence of the dividend approach to many of the other standard approaches is noted to our knowledge only in our [16] and, by implication, in Bodenhorn [1].

dividend approach has often led to the unwarranted inference that, since the investor is buying dividends and since dividend policy affects the amount of dividends, then dividend policy must also affect the current price.

Properly formulated, the dividend approach defines the current worth of a share as the discounted value of the stream of dividends to be paid on the share in perpetuity. That is

$$p(t) = \sum_{\tau=0}^{\infty} \frac{d(t+\tau)}{(1+\rho)^{\tau+1}}. \quad (13)$$

To see the equivalence between this approach and previous ones, let us first restate (13) in terms of total market value as

$$V(t) = \sum_{\tau=0}^{\infty} \frac{D_t(t+\tau)}{(1+\rho)^{\tau+1}}, \quad (14)$$

where $D_t(t+\tau)$ denotes that portion of the total dividends $D(t+\tau)$ paid during period $t+\tau$, that accrues to the shares of record as of the start of period t (indicated by the subscript). That equation (14) is equivalent to (9) and hence also to (12) is immediately apparent for the special case in which no outside financing is undertaken after period t, for in that case

$$D_t(t+\tau) = D(t+\tau) = X(t+\tau) - I(t+\tau).$$

To allow for outside financing, note that we can rewrite (14) as

$$\begin{aligned} V(t) &= \frac{1}{1+\rho}\left[D_t(t) + \sum_{\tau=1}^{\infty} \frac{D_t(t+\tau)}{(1+\rho)^{\tau}}\right] \\ &= \frac{1}{1+\rho}\left[D(t) + \sum_{\tau=0}^{\infty} \frac{D_t(t+\tau+1)}{(1+\rho)^{\tau+1}}\right]. \end{aligned} \quad (15)$$

The summation term in the last expression can be written as the difference between the stream of dividends accruing to all the shares of record as of $t+1$ and that portion of the stream that will accrue to the shares newly issued in t, that is,

$$\sum_{\tau=0}^{\infty} \frac{D_t(t+\tau+1)}{(1+\rho)^{\tau+1}} = \left(1 - \frac{m(t+1)}{n(t+1)}\right) \times \sum_{\tau=0}^{\infty} \frac{D_{t+1}(t+\tau+1)}{(1+\rho)^{\tau+1}}. \quad (16)$$

But from (14) we know that the second summation in (16) is precisely $V(t+1)$ so that (15) can be reduced to

$$\begin{aligned} V(t) &= \frac{1}{1+\rho}\left[D(t) + \left(1 - \frac{m(t+1)p(t+1)}{n(t+1)p(t+1)}\right) \times V(t+1)\right] \\ &= \frac{1}{1+\rho}\left[D(t) + V(t+1) - m(t+1)p(t+1)\right], \end{aligned} \quad (17)$$

which is (3) and which has already been shown to imply both (9) and (12).[12]

There are, of course, other ways in which the equivalence of the dividend approach to the other approaches might

[12] The statement that equations (9), (12), and (14) are equivalent must be qualified to allow for certain pathological extreme cases, fortunately of no real economic significance. An obvious example of such a case is the legendary company that is expected *never* to pay a dividend. If this were literally true then the value of the firm by (14) would be zero; by (9) it would be zero (or possibly negative since zero dividends rule out $X(t) > I(t)$ but not $X(t) < I(t)$); while by (12) the value might still be positive. What is involved here, of course, is nothing more than a discontinuity at zero since the value under (14) and (9) would be positive and the equivalence of both with (12) would hold if that value were also positive as long as there was some period T, however far in the future, beyond which the firm would pay out $\epsilon > 0$ per cent of its earnings, however small the value of ϵ.

have been established, but the method presented has the advantage perhaps of providing some further insight into the reason for the irrelevance of dividend policy. An increase in current dividends, given the firm's investment policy, must necessarily reduce the terminal value of existing shares because part of the future dividend stream that would otherwise have accrued to the existing shares must be diverted to attract the outside capital from which, in effect, the higher current dividends are paid. Under our basic assumptions, however, ρ must be the same for all investors, new as well as old. Consequently the market value of the dividends diverted to the outsiders, which is both the value of their contribution and the reduction in terminal value of the existing shares, must always be precisely the same as the increase in current dividends.

The stream of earnings approach.—Contrary to widely held views, it is also possible to develop a meaningful and consistent approach to valuation running in terms of the stream of earnings generated by the corporation rather than of the dividend distributions actually made to the shareholders. Unfortunately, it is also extremely easy to mistate or misinterpret the earnings approach as would be the case if the value of the firm were to be defined as simply the discounted sum of future total earnings.[13] The trouble with such a definition is not, as is often suggested, that it overlooks the fact that the corporation is a separate entity and that these profits cannot freely be withdrawn by the shareholders; but rather that it neglects the fact that additional capital must be acquired at some cost to maintain the future earnings stream at its specified level. The capital to be raised in any future period is, of course, $I(t)$ and its opportunity cost, no matter how financed, is ρ per cent per period thereafter. Hence, the current value of the firm under the earnings approach must be stated as

$$V(0)=\sum_{t=0}^{\infty}\frac{1}{(1+\rho)^{t+1}}\times\left[X(t)-\sum_{\tau=0}^{t}\rho I(\tau)\right]. \tag{18}$$

That this version of the earnings approach is indeed consistent with our basic assumptions and equivalent to the previous approaches can be seen by regrouping terms and rewriting equation (18) as

$$\begin{aligned}V(0)&=\sum_{t=0}^{\infty}\frac{1}{(1+\rho)^{t+1}}X(t)-\sum_{t=0}^{\infty}\left(\sum_{\tau=t}^{\infty}\frac{\rho I(t)}{(1+\rho)^{\tau+1}}\right)\\&=\sum_{t=0}^{\infty}\frac{1}{(1+\rho)^{t+1}}X(t)-\sum_{t=0}^{\infty}\frac{1}{(1+\rho)^{t+1}}\times\left(\sum_{\tau=0}^{\infty}\frac{\rho I(t)}{(1+\rho)^{\tau+1}}\right).\end{aligned} \tag{19}$$

Since the last inclosed summation reduces simply to $I(t)$, the expression (19) in turn reduces to simply

$$V(0)=\sum_{t=0}^{\infty}\frac{1}{(1+\rho)^{t+1}}[X(t)-I(t)], \tag{20}$$

[13] In fairness, we should point out that there is no one, to our knowledge, who has seriously advanced this view. It is a view whose main function seems to be to serve as a "straw man" to be demolished by those supporting the dividend view. See, e.g., Gordon [9, esp. pp. 102–3]. Other writers take as the supposed earnings counter-view to the dividend approach not a relation running in terms of the *stream* of earnings but simply the proposition that price is proportional to current earnings, i.e., $V(0)=X(0)/\rho$. The probable origins of this widespread misconception about the earnings approach are discussed further below (p. 424).

which is precisely our earlier equation (9).

Note that the version of the earnings approach presented here does not depend for its validity upon any special assumptions about the time shape of the stream of total profits or the stream of dividends per share. Clearly, however, the time paths of the two streams are closely related to each other (via financial policy) and to the stream of returns derived by holders of the shares. Since these relations are of some interest in their own right and since misunderstandings about them have contributed to the confusion over the role of dividend policy, it may be worthwhile to examine them briefly before moving on to relax the basic assumptions.

III. EARNINGS, DIVIDENDS, AND GROWTH RATES

The convenient case of constant growth rates.—The relation between the stream of earnings of the firm and the stream of dividends and of returns to the stockholders can be brought out most clearly by specializing (12) to the case in which investment opportunities are such as to generate a constant rate of growth of profits in perpetuity. Admittedly, this case has little empirical significance, but it is convenient for illustrative purposes and has received much attention in the literature.

Specifically, suppose that in each period t the firm has the opportunity to invest in real assets a sum $I(t)$ that is k per cent as large as its total earnings for the period; and that this investment produces a perpetual yield of ρ^* beginning with the next period. Then, by definition

$$\begin{aligned} X(t) &= X(t-1) + \rho^* I(t-1) \\ &= X(t-1)[1 + k\rho^*] \qquad (21) \\ &= X(0)\,[1 + k\rho^*]^t \end{aligned}$$

and $k\rho^*$ is the (constant) rate of growth of total earnings. Substituting from (21) into (12) for $I(t)$ we obtain

$$\begin{aligned} V(0) &= \frac{X(0)}{\rho} + \sum_{t=0}^{\infty}\left(\frac{\rho^* - \rho}{\rho}\right) \\ &\qquad \times kX(0)[1 + k\rho^*]^t \\ &\qquad \times (1+\rho)^{-(t+1)} \qquad (22) \\ &= \frac{X(0)}{\rho}\left[1 + \frac{k(\rho^* - \rho)}{1+\rho} \times \sum_{t=0}^{\infty}\left(\frac{1+k\rho^*}{1+\rho}\right)^t\right]. \end{aligned}$$

Evaluating the infinite sum and simplifying, we finally obtain[14]

$$\begin{aligned} V(0) &= \frac{X(0)}{\rho}\left[1 + \frac{k(\rho^* - \rho)}{\rho - k\rho^*}\right] \\ &= \frac{X(0)(1-k)}{\rho - k\rho^*}, \qquad (23) \end{aligned}$$

which expresses the value of the firm as a function of its current earnings, the rate of growth of earnings, the internal rate of return, and the market rate of return.[15]

[14] One advantage of the specialization (23) is that it makes it easy to see what is really involved in the assumption here and throughout the paper that the $V(0)$ given by any of our summation formulas is necessarily finite (cf. above, n. 4). In terms of (23) the condition is clearly $k\rho^* < \rho$, i.e., that the rate of growth of the firm be less than market rate of discount. Although the case of (perpetual) growth rates greater than the discount factor is the much-discussed "growth stock praradox" (e.g. [6]), it has no real economic significance as we pointed out in [16, esp. n. 17, p. 664]. This will be apparent when one recalls that the discount rate ρ, though treated as a constant in partial equilibrium (relative price) analysis of the kind presented here, is actually a variable from the standpoint of the system as a whole. That is, if the assumption of finite value for all shares did not hold, because for some shares $k\rho^*$ was (perpetually) greater than ρ, then ρ would necessarily rise until an over-all equilibrium in the capital markets had been restored.

[15] An interesting and more realistic variant of (22), which also has a number of convenient features from the standpoint of developing empirical tests, can be obtained by assuming that the special invest-

Note that (23) holds not just for period 0, but for every t. Hence if $X(t)$ is growing at the rate $k\rho^*$, it follows that the value of the enterprise, $V(t)$, also grows at that rate.

The growth of dividends and the growth of total profits.—Given that total earnings (and the total value of the firm) are growing at the rate $k\rho^*$ what is the rate of growth of dividends per share and of the price per share? Clearly, the answer will vary depending on whether or not the firm is paying out a high percentage of its earnings and thus relying heavily on outside financing. We can show the nature of this dependence explicitly by making use of the fact that whatever the rate of growth of dividends per share the present value of the firm by the dividend approach must be the same as by the earnings approach. Thus let

g = the rate of growth of dividends per share, or, what amounts to the same thing, the rate of growth of dividends accruing to the shares of the current holders (i.e., $D_0(t) = D_0(0)[1+g]^t$);
k_r = the fraction of total profits retained in each period (so that $D(t) = X(0)[1-k_r]$);
$k_e = k - k_r$ = the amount of external capital raised per period, expressed as a fraction of profits in the period.

Then the present value of the stream of dividends to the original owners will be

$$D_0(0)\sum_{t=0}^{\infty}\frac{(1+g)^t}{(1+\rho)^{t+1}}=\frac{D(0)}{\rho-g}=\frac{X(0)[1-k_r]}{\rho-g}. \tag{24}$$

By virtue of the dividend approach we know that (24) must be equal to $V(0)$. If, therefore, we equate it to the right-hand side of (23), we obtain

$$\frac{X(0)[1-k_r]}{\rho-g}=\frac{X(0)[1-(k_r+k_e)]}{\rho-k\rho^*}$$

from which it follows that the rate of growth of dividends per share and the rate of growth of the price of a share must be[16]

ment opportunities are available not in perpetuity but only over some finite interval of T periods. To exhibit the value of the firm for this case, we need only replace the infinite summation in (22) with a summation running from $t=0$ to $t=T-1$. Evaluating the resulting expression, we obtain

$$V(0)=\frac{X(0)}{\rho}\left\{1+\frac{k(\rho^*-\rho)}{\rho-k\rho^*}\times\left[1-\left(\frac{1+k\rho^*}{1+\rho}\right)^T\right]\right\}. \tag{22a}$$

Note that (22*a*) holds even if $k\rho^* > \rho$, so that the so-called growth paradox disappears altogether. If, as we should generally expect, $(1+k\rho^*)/(1+\rho)$ is close to one, and if T is not too large, the right hand side of (22*a*) admits of a very convenient approximation. In this case in fact we can write

$$\left[\frac{1+k\rho^*}{1+\rho}\right]^T \cong 1+T(k\rho^*-\rho)$$

the approximation holding, if, as we should expect, $(1+k\rho^*)$ and $(1+\rho)$ are both close to unity. Substituting this approximation into (22*a*) and simplifying, finally yields

$$V(0)\cong\frac{X(0)}{\rho}\left[1+\frac{k(\rho^*-\rho)}{\rho-k\rho^*}\times T(\rho-k\rho^*)\right]=\left[\frac{X(0)}{\rho}+kX(0)\times\left(\frac{\rho^*-\rho}{\rho}\right)T\right]. \tag{22b}$$

The common sense of (22*b*) is easy to see. The current value of a firm is given by the value of the earning power of the currently held assets plus the market value of the special earning opportunity multiplied by the number of years for which it is expected to last.

[16] That g is the rate of price increase per share as well as the rate of growth of dividends per share fol-

$$g = k\rho^* \frac{1-k_r}{1-k} - k_e \rho \frac{1}{1-k}. \quad (25)$$

Notice that in the extreme case in which all financing is internal ($k_e = 0$ and $k = k_r$), the second term drops out and the first becomes simply $k\rho^*$. Hence the growth rate of dividends in that special case is exactly the same as that of total profits and total value and is proportional to the rate of retention k_r. In all other cases, g is necessarily less than $k\rho^*$ and may even be negative, despite a positive $k\rho^*$, if $\rho^* < \rho$ and if the firm pays out a large fraction of its income in dividends. In the other direction, we see from (25) that even if a firm is a "growth" corporation ($\rho^* > \rho$) then the stream of dividends and price per share must grow over time even though $k_r = 0$, that is, even though it pays out *all* its earnings in dividends.

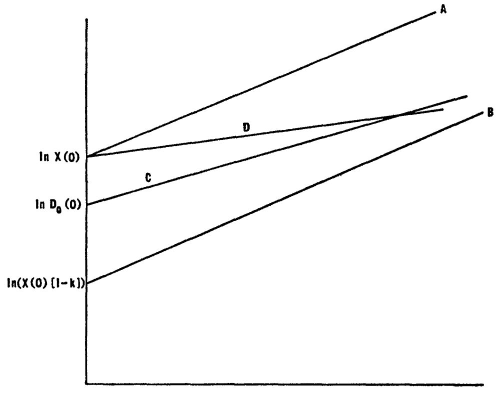

FIG. 1.—Growth of dividends per share in relation to growth in total earnings:
A. Total earnings: $\ln X(t) = \ln X(0) + k\rho^* t$;
B. Total earnings minus capital invested: $\ln [X(t) - I(t)] = \ln X(0) [1 - k] + k\rho^* t$;
Dividends per share (all financing internal): $\ln D_0(t) = \ln D(0) + gt = \ln X(0) [1 - k] + k\rho^* t$;
C. Dividends per share (some financing external): $\ln D_0(t) = \ln D(0) + gt$;
D. Dividends per share (all financing external): $\ln D_0(t) = \ln X(0) + [(k/1 - k)(\rho^* - \rho)]t$.

lows from the fact that by (13) and the definition of g

$$p(t) = \sum_{\tau=0}^{\infty} \frac{d(t+\tau)}{(1+\rho)^{\tau+1}}$$

$$= \sum_{\tau=0}^{\infty} \frac{d(0)[1+g]^{t+\tau}}{(1+\rho)^{\tau+1}}$$

$$= (1+g)^t \sum_{\tau=0}^{\infty} \frac{d(\tau)}{(1+\rho)^{\tau+1}}$$

$$= p(0)[1+g]^t.$$

The relation between the growth rate of the firm and the growth rate of dividends under various dividend policies is illustrated graphically in Figure 1 in which for maximum clarity the natural logarithm of profits and dividends have been plotted against time.[17]

Line *A* shows the total earnings of the firm growing through time at the constant rate $k\rho^*$, the slope of *A*. Line *B* shows the growth of (1) the stream of total earnings minus capital outlays and

[17] That is, we replace each discrete compounding expression such as $X(t) = X(0) [1 + k\rho^*]^t$ with its counterpart under continuous discounting $X(t) = X(0)e^{k\rho^* t}$ which, of course, yields the convenient linear relation $\ln X(t) = \ln X(0) + k\rho^* t$.

(2) the stream of dividends to the original owners (or dividends per share) in the special case in which all financing is internal. The slope of B is, of course, the same as that of A and the (constant) difference between the curves is simply $\ln(1 - k)$, the ratio of dividends to profits. Line C shows the growth of dividends per share when the firm uses both internal and external financing. As compared with the pure retention case, the line starts higher but grows more slowly at the rate g given by (25). The higher the payout policy, the higher the starting position and the slower the growth up to the other limiting case of complete external financing, Line D, which starts at $\ln X(0)$ and grows at a rate of $(k/1 - k) \cdot (\rho^* - \rho)$.

The special case of exclusively internal financing.—As noted above the growth rate of dividends per share is not the same as the growth rate of the firm except in the special case in which all financing is internal. This is merely one of a number of peculiarities of this special case on which, unfortunately, many writers have based their entire analysis. The reason for the preoccupation with this special case is far from clear to us. Certainly no one would suggest that it is the only empirically relevant case. Even if the case were in fact the most common, the theorist would still be under an obligation to consider alternative assumptions. We suspect that in the last analysis, the popularity of the internal financing model will be found to reflect little more than its ease of manipulation combined with the failure to push the analysis far enough to disclose how special and how treacherous a case it really is.

In particular, concentration on this special case appears to be largely responsible for the widely held view that, even under perfect capital markets, there is an optimum dividend policy for the firm that depends on the internal rate of return. Such a conclusion is almost inevitable if one works exclusively with the assumption, explicit or implicit, that funds for investment come *only* from retained earnings. For in that case *dividend policy* is indistinguishable from *investment policy;* and there *is* an optimal investment policy which does in general depend on the rate of return.

Notice also from (23) that if $\rho^* = \rho$ and $k = k_r$, the term $[1 - k_r]$ can be canceled from both the numerator and the denominator. The value of the firm becomes simply $X(0)/\rho$, the capitalized value of current earnings. Lacking a standard model for valuation more general than the retained earnings case it has been all too easy for many to conclude that this dropping out of the payout ratio $[1 - k_r]$ when $\rho^* = \rho$ must be what is meant by the irrelevance of dividend policy and that $V(0) = X(0)/\rho$ must constitute the "earnings" approach.

Still another example of the pitfalls in basing arguments on this special case is provided by the recent and extensive work on valuation by M. Gordon.[18] Gordon argues, in essense, that because of increasing uncertainty the discount rate $\hat{\rho}(t)$ applied by an investor to a future dividend payment will rise with t, where t denotes not a specific date but rather the distance from the period in which the investor performs the discounting.[19]

[18] See esp. [8]. Gordon's views represent the most explicit and sophisticated formulation of what might be called the "bird-in-the-hand" fallacy. For other, less elaborate, statements of essentially the same position see, among others, Graham and Dodd [11, p. 433] and Clendenin and Van Cleave [3].

[19] We use the notation $\hat{\rho}(t)$ to avoid any confusion between Gordon's purely subjective discount rate and the objective, market-given yields $\rho(t)$ in Sec. I above. To attempt to derive valuation formulas under uncertainty from these purely subjective discount factors involves, of course, an error essentially

Hence, when we use a single uniform discount rate ρ as in (22) or (23), this rate should be thought of as really an average of the "true" rates $\hat{\rho}(t)$ each weighted by the size of the expected dividend payment at time t. If the dividend stream is growing exponentially then such a weighted average ρ would, of course, be higher the greater the rate of growth of dividends g since the greater will then be the portion of the dividend stream arising in the distant as opposed to the near future. But if all financing is assumed to be internal, then $g = k_r \rho^*$ so that given ρ^*, the weighted average discount factor ρ will be an increasing function of the rate of retention k_r which would run counter to our conclusion that dividend policy has no effect on the current value of the firm or its cost of capital.

For all its ingenuity, however, and its seeming foundation in uncertainty, the argument clearly suffers fundamentally from the typical confounding of dividend policy with investment policy that so frequently accompanies use of the internal financing model. Had Gordon not confined his attention to this special case (or its equivalent variants), he would have seen that while a change in dividend policy will necessarily affect the size of the expected dividend payment on the share in any future period, it need not, in the general case, affect either the size of the *total* return that the investor expects during that period or the degree of uncertainty attaching to that total return. As should be abundantly clear by now, a change in dividend policy, given investment policy, implies a change only in the distribution of the total return in any period as between dividends and capital gains. If investors behave rationally, such a change cannot affect market valuations. Indeed, if they valued shares according to the Gordon approach and thus paid a premium for higher payout ratios, then holders of the low payout shares would actually realize consistently higher returns on their investment over any stated interval of time.[20]

Corporate earnings and investor returns.—Knowing the relation of g to $k\rho^*$ we can answer a question of considerable interest to economic theorists, namely: What is the precise relation between the earnings of the corporation in any period $X(t)$ and the total return to the owners of the stock during that period?[21] If we let $G_t(t)$ be the capital gains to the owners during t, we know that

$$D_t(t) + G_t(t) = X(t) \times (1 - k_r) + gV(t) \qquad (26)$$

analogous to that of attempting to develop the certainty formulas from "marginal rates of time preference" rather than objective market opportunities.

[20] This is not to deny that growth stocks (in our sense) may well be "riskier" than non-growth stocks. But to the extent that this is true, it will be due to the possibly greater uncertainty attaching to the size and duration of future growth opportunities and hence to the size of the future stream of total returns quite apart from any questions of dividend policy.

[21] Note also that the above analysis enables us to deal very easily with the familiar issue of whether a firm's cost of equity capital is measured by its earnings/price ratio or by its dividend/price ratio. Clearly, the answer is that it is measured by neither, except under very special circumstances. For from (23) we have for the earnings/price ratio

$$\frac{X(0)}{V(0)} = \frac{\rho - k\rho^*}{1 - k},$$

which is equal to the cost of capital ρ, only if the firm has no growth potential (i.e., $\rho^* = \rho$). And from (24) we have for the dividend/price ratio

$$\frac{D(0)}{V(0)} = \rho - g,$$

which is equal to ρ only when $g = 0$; i.e., from (25), either when $k = 0$; or, if $k > 0$, when $\rho^* < \rho$ and the amount of external financing is precisely

$$k_e = \frac{\rho^*}{\rho} k [1 - k_r],$$

so that the gain from the retention of earnings exactly offsets the loss that would otherwise be occasioned by the unprofitable investment.

since the rate of growth of price is the same as that of dividends per share. Using (25) and (26) to substitute for g and $V(t)$ and simplifying, we find that

$$D_t(t)+G_t(t)=X(t)\left[\frac{\rho(1-k)}{\rho-k\rho^*}\right]. \quad (27)$$

The relation between the investors' return and the corporation's profits is thus seen to depend entirely on the relation between ρ^* and ρ. If $\rho^* = \rho$ (i.e., the firm has no special "growth" opportunities), then the expression in brackets becomes 1 and the investor returns are precisely the same as the corporate profits. If $\rho^* < \rho$, however, the investors' return will be less than the corporate earnings; and, in the case of growth corporations the investors' return will actually be greater than the flow of corporate profits over the interval.[22]

Some implications for constructing empirical tests.—Finally the fact that we have two different (though not independent) measures of growth in $k\rho^*$ and g and two corresponding families of valuation formulas means, among other things, that we can proceed by either of two routes in empirical studies of valuation. We can follow the standard practice of the security analyst and think in terms of price per share, dividends per share, and the rate of growth of dividends per share; or we can think in terms of the total value of the enterprise, total earnings, and the rate of growth of total earnings. Our own preference happens to be for the second approach primarily because certain additional variables of interest—such as dividend policy, leverage, and size of firm—can be incorporated more easily and meaningfully into test equations in which the growth term is the growth of total earnings. But this can wait. For present purposes, the thing to be stressed is simply that two approaches, properly carried through, are in no sense *opposing* views of the valuation process; but rather equivalent views, with the choice between them largely a matter of taste and convenience.

IV. THE EFFECTS OF DIVIDEND POLICY UNDER UNCERTAINTY

Uncertainty and the general theory of valuation.—In turning now from the ideal world of certainty to one of uncertainty our first step, alas, must be to jettison the fundamental valuation principle as given, say, in our equation (3)

$$V(t)=\frac{1}{1+\rho(t)}\left[D(t)+n(t)p(t+1)\right]$$

and from which the irrelevance proposition as well as all the subsequent valua-

[22] The above relation between earnings per share and dividends plus capital gains also means that there will be a systematic relation between retained earnings and capital gains. The "marginal" relation is easy to see and is always precisely one for one regardless of growth or financial policy. That is, taking a dollar away from dividends and adding it to retained earnings (all other things equal) means an increase in capital gains of one dollar (or a reduction in capital loss of one dollar). The "average" relation is somewhat more complex. From (26) and (27) we can see that

$$G_t(t)=k_rX(t)+kX(t)\frac{\rho^*-\rho}{\rho-k\rho^*}.$$

Hence, if $\rho^* = \rho$ the total capital gain received will be exactly the same as the total retained earnings per share. For growth corporations, however, the capital gain will always be greater than the retained earnings (and there will be a capital gain of

$$kX(t)\left[\frac{\rho^*-\rho}{\rho-k\rho^*}\right]$$

even when all earnings are paid out). For non-growth corporations the relation between gain and retentions is reversed. Note also that the absolute difference between the total capital gain and the total retained earnings is a constant (given, ρ, k and ρ^*) unaffected by dividend policy. Hence the *ratio* of capital gain to retained earnings will vary directly with the payout ratio for growth corporations (and vice versa for non-growth corporations). This means, among other things, that it is dangerous to attempt to draw inferences about the relative growth potential or relative managerial efficiency of corporations solely on the basis of the ratio of capital gains to retained earnings (cf. Harkavy [12, esp. pp. 289–94]).

tion formulas in Sections II and III were derived. For the terms in the bracket can no longer be regarded as given numbers, but must be recognized as "random variables" from the point of view of the investor as of the start of period t. Nor is it at all clear what meaning can be attached to the discount factor $1/[1 + \rho(t)]$ since what is being discounted is not a given return, but at best only a probability distribution of possible returns. We can, of course, delude ourselves into thinking that we are preserving equation (3) by the simple and popular expedient of drawing a bar over each term and referring to it thereafter as the mathematical expectation of the random variable. But except for the trivial case of universal linear utility functions we know that $V(t)$ would also be affected, and materially so, by the higher order moments of the distribution of returns. Hence there is no reason to believe that the discount factor for expected values, $1/[1 + \rho(t)]$, would in fact be the same for any two firms chosen arbitrarily, not to mention that the expected values themselves may well be different for different investors.

All this is not to say, of course, that there are insuperable difficulties in the way of developing a testable theory of rational market valuation under uncertainty.[23] On the contrary, our investigations of the problem to date have convinced us that it is indeed possible to construct such a theory—though the construction, as can well be imagined, is a fairly complex and space-consuming task. Fortunately, however, this task need not be undertaken in this paper which is concerned primarily with the effects of dividend policy on market valuation. For even without a full-fledged theory of what *does* determine market value under uncertainty we can show that dividend policy at least is *not* one of the determinants. To establish this particular generalization of the previous certainty results we need only invoke a corresponding generalization of the original postulate of rational behavior to allow for the fact that, under uncertainty, choices depend on expectations as well as tastes.

"Imputed rationality" and "symmetric market rationality."—This generalization can be formulated in two steps as follows. First, we shall say that an individual trader "imputes rationality to the market" or satisfies the postulate of "imputed rationality" if, in forming expectations, he assumes that every other trader in the market is (*a*) rational in the previous sense of preferring more wealth to less regardless of the form an increment in wealth may take, and (*b*) imputes rationality to all other traders. Second, we shall say that a market as a whole satisfies the postulate of "symmetric market rationality" if every trader both behaves rationally and imputes rationality to the market.[24]

Notice that this postulate of sym-

[23] Nor does it mean that all the previous certainty analysis has no relevance whatever in the presence of uncertainty. There are many issues, such as those discussed in Sec. I and II, that really relate only to what has been called the pure "futurity" component in valuation. Here, the valuation formulas can still be extremely useful in maintaining the internal consistency of the reasoning and in suggesting (or criticizing) empirical tests of certain classes of hypotheses about valuation, even though the formulas themselves cannot be used to grind out precise numerical values for specific real-world shares.

[24] We offer the term "symmetric market rationality" with considerable diffidence and only after having been assured by game theorists that there is no accepted term for this concept in the literature of that subject even though the postulate itself (or close parallels to it) does appear frequently. In the literature of economics a closely related, but not exact counterpart is Muth's "hypothesis of rational expectations" [18]. Among the more euphonic, though we feel somewhat less revealing, alternatives that have been suggested to us are "putative rationality" (by T. J. Koopmans), "bi-rationality" (by G. L. Thompson), "empathetic rationality" (by Andrea Modigliani), and "panrationality" (by A. Ando).

metric market rationality differs from the usual postulate of rational behavior in several important respects. In the first place, the new postulate covers not only the choice behavior of individuals but also their expectations of the choice behavior of others. Second, the postulate is a statement about the market as a whole and not just about individual behavior. Finally, though by no means least, symmetric market rationality cannot be deduced from individual rational behavior in the usual sense since that sense does not imply imputing rationality to others. It may, in fact, imply a choice behavior inconsistent with imputed rationality unless the individual actually believes the market to be symmetrically rational. For if an ordinarily rational investor had good reason to believe that other investors would not behave rationally, then it might well be rational for him to adopt a strategy he would otherwise have rejected as irrational. Our postulate thus rules out, among other things, the possibility of speculative "bubbles" wherein an individually rational investor buys a security he knows to be overpriced (i.e., too expensive in relation to its expected *long-run* return to be attractive as a permanent addition to his portfolio) in the expectation that he can resell it at a still more inflated price before the bubble bursts.[25]

The irrelevance of dividend policy despite uncertainty.—In Section I we were able to show that, given a firm's investment policy, its dividend policy was irrelevant to its current market valuation. We shall now show that this fundamental conclusion need not be modified merely because of the presence of uncertainty about the future course of profits, investment, or dividends (assuming again, as we have throughout, that investment policy can be regarded as separable from dividend policy). To see that uncertainty about these elements changes nothing essential, consider a case in which current investors believe that the future streams of total earnings and total investment whatever actual values they may assume at different points in time will be identical for two firms, 1 and 2.[26] Suppose further, provisionally, that the same is believed to be true of future total dividend payments from period one on so that the only way in which the two firms differ is possibly with respect to the prospective dividend in the current period, period 0. In terms of previous notation we are thus assuming that

$$\tilde{X}_1(t) = \tilde{X}_2(t) \qquad t = 0 \ldots \infty$$
$$\tilde{I}_1(t) = \tilde{I}_2(t) \qquad t = 0 \ldots \infty$$
$$\tilde{D}_1(t) = \tilde{D}_2(t) \qquad t = 1 \ldots \infty$$

[25] We recognize, of course, that such speculative bubbles have actually arisen in the past (and will probably continue to do so in the future), so that our postulate can certainly not be taken to be of universal applicability. We feel, however, that it is also not of universal inapplicability since from our observation, speculative bubbles, though well publicized when they occur, do not seem to us to be a dominant, or even a fundamental, feature of actual market behavior under uncertainty. That is, we would be prepared to argue that, as a rule and on the average, markets do not behave in ways which do not obviously contradict the postulate so that the postulate may still be useful, at least as a first approximation, for the analysis of long-run tendencies in organized capital markets. Needless to say, whether our confidence in the postulate is justified is something that will have to be determined by empirical tests of its implications (such as, of course, the irrelevance of dividend policy).

[26] The assumption of two identical firms is introduced for convenience of exposition only, since it usually is easier to see the implications of rationality when there is an explicit arbitrage mechanism, in this case, switches between the shares of the two firms. The assumption, however, is not necessary and we can, if we like, think of the two firms as really corresponding to two states of the same firm for an investor performing a series of "mental experiments" on the subject of dividend policy.

the subscripts indicating the firms and the tildes being added to the variables to indicate that these are to be regarded from the standpoint of current period, not as known numbers but as numbers that will be drawn in the future from the appropriate probability distributions. We may now ask: "What will be the return, $\tilde{R}_1(0)$ to the current shareholders in firm 1 during the current period?" Clearly, it will be

$$\tilde{R}_1(0) = \tilde{D}_1(0) + \tilde{V}_1(1) - \tilde{m}_1(1)\,\tilde{p}_1(1)\,. \quad (28)$$

But the relation between $\tilde{D}_1(0)$ and $\tilde{m}_1(1)\,\tilde{p}_1(1)$ is necessarily still given by equation (4) which is merely an accounting identity so that we can write

$$\tilde{m}_1(1)\,\tilde{p}_1(1) = \tilde{I}_1(0) - [\tilde{X}_1(0) - \tilde{D}_1(0)], \quad (29)$$

and, on substituting in (28), we obtain

$$\tilde{R}_1(0) = \tilde{X}_1(0) - \tilde{I}_1(0) + \tilde{V}_1(1) \quad (30)$$

for firm 1. By an exactly parallel process we can obtain an equivalent expression for $\tilde{R}_2(0)$.

Let us now compare $\tilde{R}_1(0)$ with $\tilde{R}_2(0)$. Note first that, by assumption, $\tilde{X}_1(0) = \tilde{X}_2(0)$ and $\tilde{I}_1(0) = \tilde{I}_2(0)$. Furthermore, with symmetric market rationality, the terminal values $\tilde{V}_i(1)$ can depend only on prospective future earnings, investment and dividends from period 1 on and these too, by assumption, are identical for the two companies. Thus symmetric rationality implies that every investor must expect $\tilde{V}_1(1) = \tilde{V}_2(1)$ and hence finally $\tilde{R}_1(0) = \tilde{R}_2(0)$. But if the return to the investors is the same in the two cases, rationality requires that the two firms command the same current value so that $V_1(0)$ must equal $V_2(0)$ regardless of any difference in dividend payments during period 0. Suppose now that we allow dividends to differ not just in period 0 but in period 1 as well, but still retain the assumption of equal $\tilde{X}_i(t)$ and $\tilde{I}_i(t)$ in all periods and of equal $\tilde{D}_i(t)$ in period 2 and beyond. Clearly, the only way differences in dividends in period 1 can effect $\tilde{R}_i(0)$ and hence $V_i(0)$ is via $\tilde{V}_i(1)$. But, by the assumption of symmetric market rationality, current investors know that as of the start of period 1 the then investors will value the two firms rationally and we have already shown that differences in the current dividend do not affect current value. Thus we must have $\tilde{V}_1(1) = \tilde{V}_2(1)$—and hence $V_1(0) = V_2(0)$—regardless of any possible difference in dividend payments during period 1. By an obvious extension of the reasoning to $\tilde{V}_i(2)$, $\tilde{V}_i(3)$, and so on, it must follow that the current valuation is unaffected by differences in dividend payments in *any* future period and thus that dividend policy is irrelevant for the determination of market prices, given investment policy.[27]

Dividend policy and leverage.—A study of the above line of proof will show it to be essentially analogous to the proof for the certainty world, in which as we know, firms can have, in effect, only two alternative sources of investment funds: retained earnings or stock issues. In an uncertain world, however, there is the additional financing possibility of debt issues. The question naturally arises, therefore, as to whether the conclusion about irrelevance remains valid even in the presence of debt financing, particularly since there may very well be inter-

[27] We might note that the assumption of symmetric market rationality is sufficient to derive this conclusion but not strictly necessary if we are willing to weaken the irrelevance proposition to one running in terms of long-run, average tendencies in the market. Individual rationality alone could conceivably bring about the latter, for over the long pull rational investors could enforce this result by buying and holding "undervalued" securities because this would insure them higher long-run returns when eventually the prices became the same. They might, however, have a long, long wait.

actions between debt policy and dividend policy. The answer is that it does, and while a complete demonstration would perhaps be too tedious and repetitious at this point, we can at least readily sketch out the main outlines of how the proof proceeds. We begin, as above, by establishing the conditions from period 1 on that lead to a situation in which $\tilde{V}_1(1)$ must be brought into equality with $\tilde{V}_2(1)$ where the V, following the approach in our earlier paper [17], is now to be interpreted as the total market value of the firm, debt plus equity, not merely equity alone. The return to the original investors taken as a whole—and remember that any individual always has the option of buying a proportional share of both the equity and the debt—must correspondingly be broadened to allow for the interest on the debt. There will also be a corresponding broadening of the accounting identity (4) to allow, on the one hand, for the interest return and, on the other, for any debt funds used to finance the investment in whole or in part. The net result is that both the dividend component and the interest component of total earnings will cancel out making the relevant (total) return, as before, $[\tilde{X}_i(0) - \tilde{I}_i(0) + \tilde{V}_i(1)]$ which is clearly independent of the current dividend. It follows, then, that the value of the firm must also therefore be independent of dividend policy given investment policy.[28]

The informational content of dividends.—To conclude our discussion of dividend policy under uncertainty, we might take note briefly of a common confusion about the meaning of the irrelevance proposition occasioned by the fact that in the real world a change in the dividend rate is often followed by a change in the market price (sometimes spectacularly so). Such a phenomenon would not be incompatible with irelevance to the extent that it was merely a reflection of what might be called the "informational content" of dividends, an attribute of particular dividend payments hitherto excluded by assumption from the discussion and proofs. That is, where a firm has adopted a policy of dividend stabilization with a long-established and generally appreciated "target payout ratio," investors are likely to (and have good reason to) interpret a change in the dividend rate as a change in management's views of future profit prospects for the firm.[29] The dividend change, in other words, provides the occasion for the price change though not its cause, the price still being solely a reflection of future earnings and growth opportunities. In any particular instance, of course, the investors might well be mistaken in placing this interpretation on the dividend change, since the management might really only be changing ing its payout target or possibly even attempting to "manipulate" the price. But this would involve no particular conflict with the irrelevance proposition, unless, of course, the price changes in such cases were not reversed when the unfolding of events had made clear the true nature of the situation.[30]

[28] This same conclusion must also hold for the current market value of all the shares (and hence for the current price per share), which is equal to the total market value minus the given initially outstanding debt. Needless to say, however, the price per share and the value of the equity at *future* points in time will not be independent of dividend and debt policies in the interim.

[29] For evidence on the prevalence of dividend stabilization and target ratios see Lintner [15].

[30] For a further discussion of the subject of the informational content of dividends, including its implications for empirical tests of the irrelevance proposition, see Modigliani and Miller [16, pp. 666–68].

V. DIVIDEND POLICY AND MARKET IMPERFECTIONS

To complete the analysis of dividend policy, the logical next step would presumably be to abandon the assumption of perfect capital markets. This is, however, a good deal easier to say than to do principally because there is no unique set of circumstances that constitutes "imperfection." We can describe not one but a multitude of possible departures from strict perfection, singly and in combinations. Clearly, to attempt to pursue the implications of each of these would only serve to add inordinately to an already overlong discussion. We shall instead, therefore, limit ourselves in this concluding section to a few brief and general observations about imperfect markets that we hope may prove helpful to those taking up the task of extending the theory of valuation in this direction.

First, it is important to keep in mind that from the standpoint of dividend policy, what counts is not imperfection per se but only imperfection that might lead an investor to have a systematic preference as between a dollar of current dividends and a dollar of current capital gains. Where no such systematic preference is produced, we can subsume the imperfection in the (random) error term always carried along when applying propositions derived from ideal models to real-world events.

Second, even where we do find imperfections that bias individual preferences—such as the existence of brokerage fees which tend to make young "accumulators" prefer low-payout shares and retired persons lean toward "income stocks"—such imperfections are at best only necessary but not sufficient conditions for certain payout policies to command a permanent premium in the market. If, for example, the frequency distribution of corporate payout ratios happened to correspond exactly with the distribution of investor preferences for payout ratios, then the existence of these preferences would clearly lead ultimately to a situation whose implications were different in no fundamental respect from the perfect market case. Each corporation would tend to attract to itself a "clientele" consisting of those preferring its particular payout ratio, but one clientele would be entirely as good as another in terms of the valuation it would imply for the firm. Nor, of course, is it necessary for the distributions to match exactly for this result to occur. Even if there were a "shortage" of some particular payout ratio, investors would still normally have the option of achieving their particular saving objectives without paying a premium for the stocks in short supply simply by buying appropriately weighted combinations of the more plentiful payout ratios. In fact, given the great range of corporate payout ratios known to be available, this process would fail to eliminate permanent premiums and discounts only if the distribution of investor preferences were heavily concentrated at either of the extreme ends of the payout scale.[31]

Of all the many market imperfections that might be detailed, the only one that would seem to be even remotely capable of producing such a concentration is the substantial advantage accorded to capital gains as compared with dividends un-

[31] The above discussion should explain why, among other reasons, it would not be possible to draw any valid inference about the relative preponderance of "accumulators" as opposed to "income" buyers or the strength of their preferences merely from the weight attaching to dividends in a simple cross-sectional regression between value and payouts (as is attempted in Clendenin [2, p. 50] or Durand [5, p. 651]).

der the personal income tax. Strong as this tax push toward capital gains may be for high-income individuals, however, it should be remembered that a substantial (and growing) fraction of total shares outstanding is currently held by investors for whom there is either no tax differential (charitable and educational institutions, foundations, pension trusts, and low-income retired individuals) or where the tax advantage is, if anything, in favor of dividends (casualty insurance companies and taxable corporations generally). Hence, again, the "clientele effect" will be at work. Furthermore, except for taxable individuals in the very top brackets, the required difference in before-tax yields to produce equal after-tax yields is not particularly striking, at least for moderate variations in the composition of returns.[32] All this is not to say, of course, that differences in yields (market values) caused by differences in payout policies should be ignored by managements or investors merely because they may be relatively small. But it may help to keep investigators from being too surprised if it turns out to be hard to measure or even to detect any premium for low-payout shares on the basis of standard statistical techniques.

Finally, we may note that since the tax differential in favor of capital gains is undoubtedly the major *systematic* imperfection in the market, one clearly cannot invoke "imperfections" to account for the difference between our irrelevance proposition and the standard view as to the role of dividend policy found in the literature of finance. For the standard view is not that low-payout companies command a premium; but that, in general, they will sell at a discount![33] If such indeed were the case—and we, at least, are not prepared to concede that this has been established—then the analysis presented in this paper suggests there would be only one way to account for it; namely, as the result of systematic irrationality on the part of the investing public.[34]

To say that an observed positive premium on high payouts was due to irrationality would not, of course, make the phenomenon any less real. But it would at least suggest the need for a certain measure of caution by long-range policymakers. For investors, however naïve they may be when they enter the market, do sometimes learn from experience; and perhaps, occasionally, even from reading articles such as this.

[32] For example, if a taxpayer is subject to a marginal rate of 40 per cent on dividends and half that or 20 per cent on long-term capital gains, then a before-tax yield of 6 per cent consisting of 40 per cent dividends and 60 per cent capital gains produces an after-tax yield of 4.32 per cent. To net the same after-tax yield on a stock with 60 per cent of the return in dividends and only 40 per cent in capital gains would require a before-tax yield of 6.37 per cent. The difference would be somewhat smaller if we allowed for the present dividend credit, though it should also be kept in mind that the tax on capital gains may be avoided entirely under present arrangements if the gains are not realized during the holder's lifetime.

[33] See, among many, many others, Gordon [8, 9], Graham and Dodd [11, esp. chaps. xxxiv and xxxvi], Durand [4, 5], Hunt, Williams, and Donaldson [13, pp. 647–49], Fisher [7], Gordon and Shapiro [10], Harkavy [12], Clendenin [2], Johnson, Shapiro, and O'Meara [14], and Walter [19].

[34] Or, less plausibly, that there is a systematic tendency for external funds to be used more productively than internal funds.

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